

HARMONIC MOTION OF CYLINDERS  
IN UNIFORM FLOW

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# THESIS

Harmonic Motion of  
Cylinders in Uniform Flow

by

James Thomas Fry

June 1975

Thesis Advisor:

T. Sarpkaya

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## 20. Abstract (continued)

Fourier-averaged drag and inertia coefficients  $C_d$  and  $C_m$  as functions of the relative amplitude  $A/D$  and the frequency parameter  $D/\bar{V}T$ .







Harmonic Motion of  
Cylinders in Uniform Flow

by

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## ABSTRACT

The time-dependent force acting on a cylinder undergoing harmonic in-line oscillations in an otherwise steady flow was measured for various amplitudes and frequencies of oscillation and mean flow velocity. The experiments were carried out in a recirculating water tunnel operating as an open channel with a free surface at the test section. The time-dependent force has been expressed in terms of a mean drag coefficient  $\bar{C}_d$  and the Fourier-averaged drag and inertia coefficients  $C_d$  and  $C_m$  as functions of the relative amplitude  $A/D$  and the frequency parameter  $D/\bar{V}T$ .



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## NOMENCLATURE

$A$	amplitude of cylinder oscillation
$\bar{C}_d$	mean drag coefficient
$C_d$	Fourier-averaged drag coefficient
$C_m$	Fourier-averaged inertia coefficient
$D$	diameter of test cylinder
$F$	instantaneous total force acting on the test cylinder
$Re$	Reynolds number ( $Re = \bar{V}D/\nu$ )
$T$	period of oscillation
$t$	time
$U$	instantaneous velocity
$U_m$	maximum velocity in a cycle ( $U_m = 2\pi A/T$ )
$\bar{V}$	mean velocity of the uniform flow
$\theta$	phase angle
$\nu$	fluid kinematic viscosity
$\rho$	fluid density



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## I. INTRODUCTION

Elastic structures of one or more degrees of freedom can extract energy from the flow about them and can develop catastrophic flow-induced vibrations. The understanding of this energy-extraction process is of paramount importance if one is either to eliminate or minimize it or to design the elastic structure such that it can withstand the oscillations under the contemplated environmental conditions.

Even a superficial familiarity with the parameters involved in this type of phenomenon shows that the investigation of the flow-induced vibrations or the flow about oscillating bodies is more than an extension of the past studies on flow about bluff bodies at rest. In fact, the complexity of the problem is increased by an order of magnitude. In view of this fact, it is no wonder that the past decade has produced either numerous experimental data with the goal of obtaining ad-hoc solutions for specific problems, while directing little if any attention toward elucidation of the underlying mechanics and toward generalization of the results for future applications; or numerous analytical models most or all of which had nothing to do with the motion of the very medium which provided the necessary energy to set the body in motion. In fact, the fluid mechanics of the flow-induced oscillations became so





incidental to the phenomenon that the models dealt essentially with black-box-induced oscillations. To be sure, these models heavily relied on experimental data partly to justify their existence and partly to assign numerical values to numerous variable constants imbedded in them. The ability of these models to scale, i.e., to permit extrapolation, is uncertain. Thus, in the final analysis one is not quite sure whether one should use the experimental data within the range of their application or the curves fitted to them by the empirical models. Suffice it to say that the phenomenon is far from understood even for the idealized conditions encountered in the laboratory without the alarming consequences of the real ocean environment where fauna and flora, ever-changing ocean currents, and temperature gradients add further complications to an already complex problem. Surely, over design is not the answer but it may, in the next decade or so, be the best available tradeoff with failure.

Mathematical models of flow-induced vibrations of bluff bodies and the response of circular cylinders to vortex shedding have been aptly described by Parkinson [1] and Currie, et al. [2] and will not be repeated here.

It appears that among the various models considered so far, the "wake-oscillator" model of Hartlen and Currie [3] attracted more attention among the students of vibration analysis. We will not elaborate here on the attempts made by others to introduce one or more additional terms into the



model originally proposed by Hartlen and Currie for such naive attempts produced only more papers and unrealistically defined coefficients than sound information toward the understanding of the flow-induced oscillations both in-line with and transverse to the ambient flow.

The in-line oscillations has been subjected to very little investigation. Chen and Ballengee [4] examined the vortex shedding from circular cylinders in an oscillating freestream of 3 Hz with  $A/D$  from 15 to 1,000,  $D/\bar{V}T = 0.003$ , and the Reynolds numbers up to 40,000, the vortex shedding from a circular cylinder responds instantaneously to the freestream variations and that "the instantaneous Strouhal number stays sensibly constant at  $0.2 \pm 0.01$ ." This is rather expected since the amplitude of oscillations is many times that of the cylinder diameter and the flow in the absence of significant accelerations exhibits a quasi-steady behavior.

Hatfield and Morkovin [5] studied the effect of an oscillating freestream on the unsteady pressure on a circular cylinder ( $D/\bar{V}T$  from 0.15 to 0.25,  $A/D$  from 0.05 to 0.087, and  $Re = 50,000$ ). They have found that there is no significant coupling between the small-amplitude freestream oscillations and the vortex shedding. Their results would suggest that the drag coefficient associated with the mean flow would essentially remain constant at its steady state value. This study, unlike the previous ones, concentrated



on the other extreme of the  $A/D$  values. In this case, the flow fluctuations in the neighborhood of the structure are essentially uncorrelated and therefore relatively ineffective. Evidently, for values of  $D/\bar{V}T$  from about 0.1 to 0.5 and  $A/D$  from about 0.2 to 1.0 that the interaction of oscillations with the body becomes significant and rather complex. The study of the two extremes of  $D/\bar{V}T$  and  $A/D$  does not shed much light on the understanding of oscillating flow about bluff bodies.

Mercier [6] who subjected cylinders to large streamwise oscillations found that the averaging drag coefficient significantly increases with  $D/\bar{V}T$  and that the rate of increase depends on the amplitude to diameter ratio in the range of  $A/D$  from 0.2 to 3.0,  $nD/\bar{V}$  from 0.1 to infinity, and  $Re$  from 4,000 to 16,000.

Davenport [7] subjected bluff bodies (flat, plate, circular and triangular cylinders, and lattice truss) to small amplitude oscillations in a water flume and evaluated the drag and inertia coefficients through the measurement of the rate of damping of the amplitude of oscillations of the bodies, i.e., without directly measuring the forces acting on the bodies. The experiments were carried out in the range of  $D/\bar{V}T$  from 0.02 to 2.0. The Reynolds number ranged from about 1700 to 8,000. Davenport evaluated  $C_m$  and  $C_d$  assuming  $\bar{C}_d$  to be equal to that for the corresponding steady flow at the velocity  $\bar{V}$  and ignoring the term



involving  $U_m^2$ . Since  $\bar{C}_d$  does not remain constant as evidenced by the present study and  $U_m^2$  cannot be neglected for  $U_m/\bar{V}$  larger than about 0.1, Davenport's  $C_d$  and  $C_m$  values are not comparable with those presented herein.

In a related study, Tseng [8] conducted experiments with flat plates normal to the stream undergoing freely decaying oscillations. His results have shown that the presence of the mean flow significantly increases the damping force and that the rate of extinction of the oscillation increases monotonically with speed.

Tanida, Okajima, and Watanabe [9] found that for a circular cylinder oscillating parallel to the flow ( $A/D = 0.14$ ,  $D/\bar{V}T$  from zero to 0.5, and for  $Re = 80$  and 4,000), the vortex synchronization can be observed in a range around double the Strouhal frequency, where vortices are shed with a frequency half the imposed one. Tanida, et al.'s results show that the mean drag reaches its maximum in the middle of the synchronization range, i.e.,  $D/\bar{V}T$  between 0.2 and 0.4, and that the sign of  $C_d$  is such that no energy can be extracted from the fluid to render the oscillations unstable. In other words, in-line oscillation is stable for the two Reynolds numbers tested. Tanida conjectured that instability is likely to occur at much higher Reynolds numbers. Unfortunately, their  $C_d$  values cannot be relied upon since the inertial forces were subtracted only approximately and, according to the data reported herein,





are far from being correct. Thus, their  $C_d$  values cannot be considered sufficiently accurate to assess the stability of in-line oscillations.

Goddard [10] carried out a numerical solution of the drag response of a cylinder to streamwise velocity fluctuations for  $Re = 40$  and  $nD/\bar{V} = 0.019, 0.12, \text{ and } 3.18$ , and for  $Re = 200$  and  $nD/\bar{V} = 0.149$ . He found that for very low frequencies the instantaneous values of drag correspond very nearly to the quasi-steady solution and that for higher frequencies the drag anticipates the freestream velocity maximum. This work cannot be generalized to higher Reynolds numbers since it is based on the Navier-Stokes equations and since the diffusion of vorticity in the concentrated vortices for  $Re$  larger than about 200 is primarily turbulent. Numerical or exact solutions of the Navier-Stokes equations cannot take into consideration such a turbulent diffusion unless modified through the use of an appropriate eddy viscosity.

#### DECOMPOSITION OF MEASURED FORCES

The numerical calculations as well as measurements in time-dependent flow yield the resultant force as a function of time for a given set of numerical values of the independent parameters. Thus it is not possible without a suitable hypothesis, to express the force both as a function of time and remaining independent parameters as one ordinarily would in a closed form solution. Such a working hypothesis is



particularly necessary for the cable strumming problem since the results are to be incorporated into the dynamics of the cylinder motion in the form of a forcing function. It should be stated at the outset that there is, at present, no generally accepted hypothesis to decompose the time-dependent force into suitable components. As it will be seen shortly, even the existing hypotheses, such as the so-called Morison's equations, are not applicable to periodic flows with a non-zero mean velocity.

Stokes, in a remarkable paper on the motion of pendulums, showed that the expression for the force on a sphere oscillation in an unlimited viscous fluid consists of two terms, one involving the acceleration of the sphere and the other the velocity. This analysis shows that the inertia coefficient is modified because of viscosity and is augmented over the theoretical value valid for irrotational flow. The drag coefficient associated with the velocity is modified because of acceleration, and its value is greater than it would be if the sphere were moving with a constant velocity. In general, the force experienced by a bluff body at a given time depends on the entire history of its acceleration as well as the instantaneous velocity and acceleration. Thus, the drag coefficient in unsteady flow is not equal to that at the same instantaneous velocity in steady flow. Neither is the inertia coefficient equal to that found for unseparated potential flow. As yet a theoretical analysis of the problem



for separated flow is difficult and much of the desired information must be obtained both experimentally and numerically. In this respect, the experimental studies of Morison and his co-workers [11] on the forces on piles due to the action of progressive waves have shed considerable light on the problem. The forces are divided into two parts, one due to the drag, as in the case of flow of constant velocity, and the other due to the acceleration or deceleration of the fluid. This concept necessitates the introduction of a drag coefficient  $C_d$  and an inertia coefficient  $C_m$  in the expression for force. In particular if  $F$  is the force per unit length experienced by a cylinder, then

$$F = 0.5 C_d \rho D U |U| + C_m \rho \pi D^2 / 4 dU/dt$$

where  $U$  and  $dU/dt$  represent respectively the undisturbed velocity and the acceleration of the fluid.

On the basis of irrotational flow around the cylinder,  $C_m$  should be equal to 2 (cylinder at rest, the fluid accelerating; otherwise  $C_m = 1$ ), and one may suppose that the value of  $C_d$  should be identical with that applicable to a constant velocity. However, numerous experiments show that this is not the case and that  $C_d$  and  $C_m$  show considerable variations from those just cited above. Even though no one has suggested a better alternative, the use of the Morison's equation gave rise to a great deal of discussion on what





values of the two coefficients should be used. Furthermore, the importance of the viscosity effect has remained in doubt since the experimental evidence published over the said period has been quite inconclusive.

The drag and inertia coefficients obtained from a large number of field tests, as compiled by Wiegel [12], show extensive scatter whether they are plotted as a function of the Reynolds number or the so-called period parameter  $U_m T/D$ . The reasons for the observed scatter of the coefficients  $C_m$  and  $C_d$  remained largely unknown. The scatter was attributed to several reasons or combinations thereof such as the irregularity of the ocean waves, free-surface effects, inadequacy of the average resistance coefficients to represent the actual variation of the nonlinear force, omission of some other important parameter which has not been incorporated into the analysis, the effect of ocean currents on separation, vortex formation, and hence on the forces acting on the cylinders, etc.

The most systematic evaluation of the Fourier-averaged drag and inertia coefficients has been made by Keulegan and Carpenter [3] through measurements on submerged horizontal cylinders and plates in the node of a standing wave, applying theoretically derived values for velocities and accelerations. Additional measurements have been made by Sarpkaya [14] of the in-lines as well as transverse forces acting on cylinders and spheres in a sinusoidally oscillating fluid and it was



found that the drag coefficient as well as the inertia coefficient for a strictly sinusoidally oscillating fluid (no mean velocity) is a function of  $U_m T/D$  and that the effect of the Reynolds number is rather secondary and certainly obscured by the excellent correlation of the data with the period parameter  $U_m T/D$ .

On the basis of the above discussion, one would assume that Morison's equation would apply equally well to periodic flow with a mean velocity where  $u = \bar{V} - U_m \cos \theta$  and that  $C_d$  and  $C_m$  will have constant, time-invariant, Fourier or least-squares averages. This, in turn, implies that  $C_d$  and  $C_m$  are independent of the associated flow phenomena. There is, however, no a priori assurance in the principles of fluid mechanics or theory of models that this is, in fact, the case. Thus the effect of the combination of a uniform current and harmonic oscillations on the time-average and oscillatory forces acting on circular cylinders will have to be re-examined and the limits of application of the Morison's equation be delineated.

It is a priori evident that both  $u = -U_m \cos \theta$  and  $u = \bar{V} - U_m \cos \theta$  yield the same acceleration  $du/dt$ . Thus, the force in-phase with the acceleration in Morison's equation remains unaffected by the presence of the mean flow. The results presented herein show that this is not the case. Furthermore, the use of the Morison equation as



$$F = 0.5\rho C_d(\bar{V} - U_m \cos \theta) |V - U_m \cos \theta| + C_m \rho \pi D^2/4 \cdot du/dt$$

requires that the time-averaged drag force be calculated by increasing the force calculated from the steady flow by a factor  $[1 + 0.5(U_m/\bar{V})^2]$ . The results presented herein show that such an analysis appreciably underestimates the measured mean forces. It suffices to state that the fluid flow phenomena for bluff bodies are significantly affected by the combination of currents and harmonic oscillations and that the results for steady currents alone and oscillations alone cannot be combined to yield reliable estimates of forces due to both acting together.

The time-dependent forces in the present study are analyzed according to the following three-coefficient equation

$$F = 0.5\bar{C}_d \rho D \bar{V}^2 + C_m \rho \frac{\pi D^2}{4} \frac{d}{dt}(-U_m \cos \frac{2\pi}{T}t) - \\ C_d D U_m^2 \frac{1}{2} |\cos \frac{2\pi}{T}t| \cos \frac{2\pi}{T}t$$

which may be written as,

$$\frac{F}{0.5\rho D \bar{V}^2} = \bar{C}_d + C_m \pi^2 (U_m T/D) (D/\bar{V}T)^2 \sin \frac{2\pi}{T}t - \\ C_d (U_m T/D)^2 (D/\bar{V}T)^2 |\cos \frac{2\pi}{T}t| \cos \frac{2\pi}{T}t$$



in which  $C_m$  and  $C_d$  are given by their Fourier averages as

$$C_m = (2U_m T / \pi^3 D) \int_0^{2\pi} (F \sin \theta) d\theta / (\rho U_m^2 D)$$

and

$$C_d = -(3/4) \int_0^{2\pi} (F \sin \theta) d\theta / (\rho U_m^2 D)$$

Evidently,  $C_d$ ,  $C_m$ , and  $\bar{C}_d$  are functions of  $\bar{V}T/D$  and  $U_m T/D$  or  $A/D$ . They may depend also on the Reynolds number which does not explicitly appear in the above expression because of the assumptions made in the formulation of the basic force equation.

In the foregoing, neither the coefficient  $\bar{C}_d$  is assumed to be equal to the steady-state drag coefficient for a uniform flow at the constant velocity  $\bar{V}$ , nor  $C_m$  and  $C_d$  are assumed to be identical to those obtained for a strictly harmonic oscillation. In fact, the results show that  $\bar{C}_d = C_d$  (steady) only for  $U_m = 0$ , and  $C_d$  and  $C_m$  are equal to those obtained for the harmonic oscillation only for  $\bar{V}T/D = 0$ .





## II. EXPERIMENTAL APPARATUS AND PROCEDURE

### A. EQUIPMENT

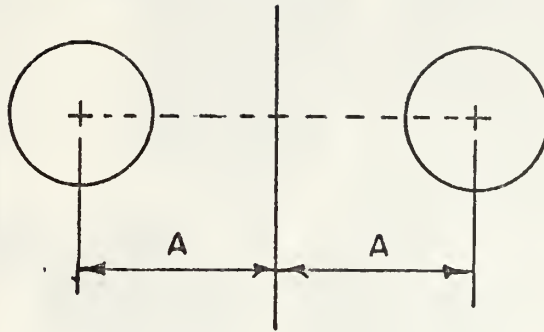
#### 1. NPS Water Tunnel

The experiments were performed in a recirculating water tunnel (Figure 1) which had a capacity of approximately 500 gallons. The galvanized test section was four inches wide, eight inches high, and sixteen inches long. A low-rpm, high-capacity, fourteen-inch-diameter-discharge centrifugal pump was used to circulate the fluid through the test section. The velocity of the fluid was regulated by a butterfly valve arrangement which was situated downstream of the test specimen. Velocities of .7 to 1.5 fps were obtained by adjusting the vains of the butterfly valve.

#### 2. Pendulum and Motor Arrangement

The harmonic motion in this experiment was obtained by a small, variable speed, electric motor and crank arrangement (Figure 2). The crank was attached to the arm of a pendulum 41 inches from the pivot point. The overall length of the pendulum was 56 inches. The frequency of the pendulum oscillation was regulated by the speed of the electric motor. The frequency of oscillation varied from 0.6 to 9.0 c.p.s. The amplitude of the oscillation was set by adjusting the end of the crank on a rotating disk (Figure 2). The amplitude "A", ranged from  $3/16$  to 1.5 inches.





### 3. Test Specimen

Circular cylinders of 3/4 inch and one inch were used as the test specimen. The 3/4 inch cylinder was made of plexiglass and the one inch cylinder was made of aluminum tubing. The test specimen was switched to aluminum tubing because the inertial force experienced was less than that experienced with the plexiglass cylinder.

The cylinders were attached to the arm of the pendulum by means of an aluminum yoke arrangement (Figure 3). The action of the pendulum caused the cylinder to oscillate in the center of the test section and in-line with the stream flow.

In order to oscillate the cylinder with the pendulum arrangement, it was necessary to operate the tunnel with the top of the test section open. The water level was maintained as close to the top of the test section as possible in order to make the free surface effects negligible.





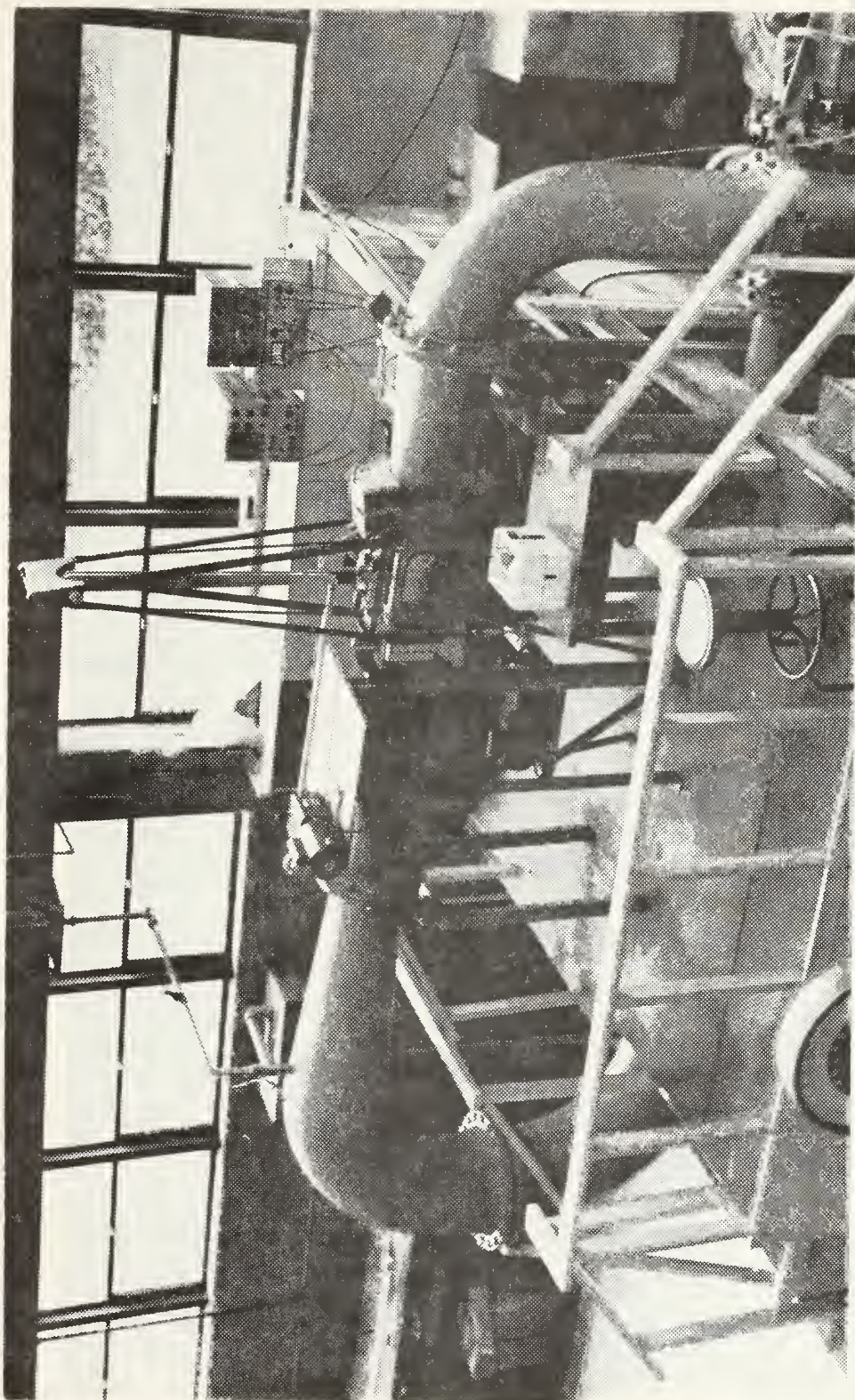


Figure 1. NPS water tunnel.





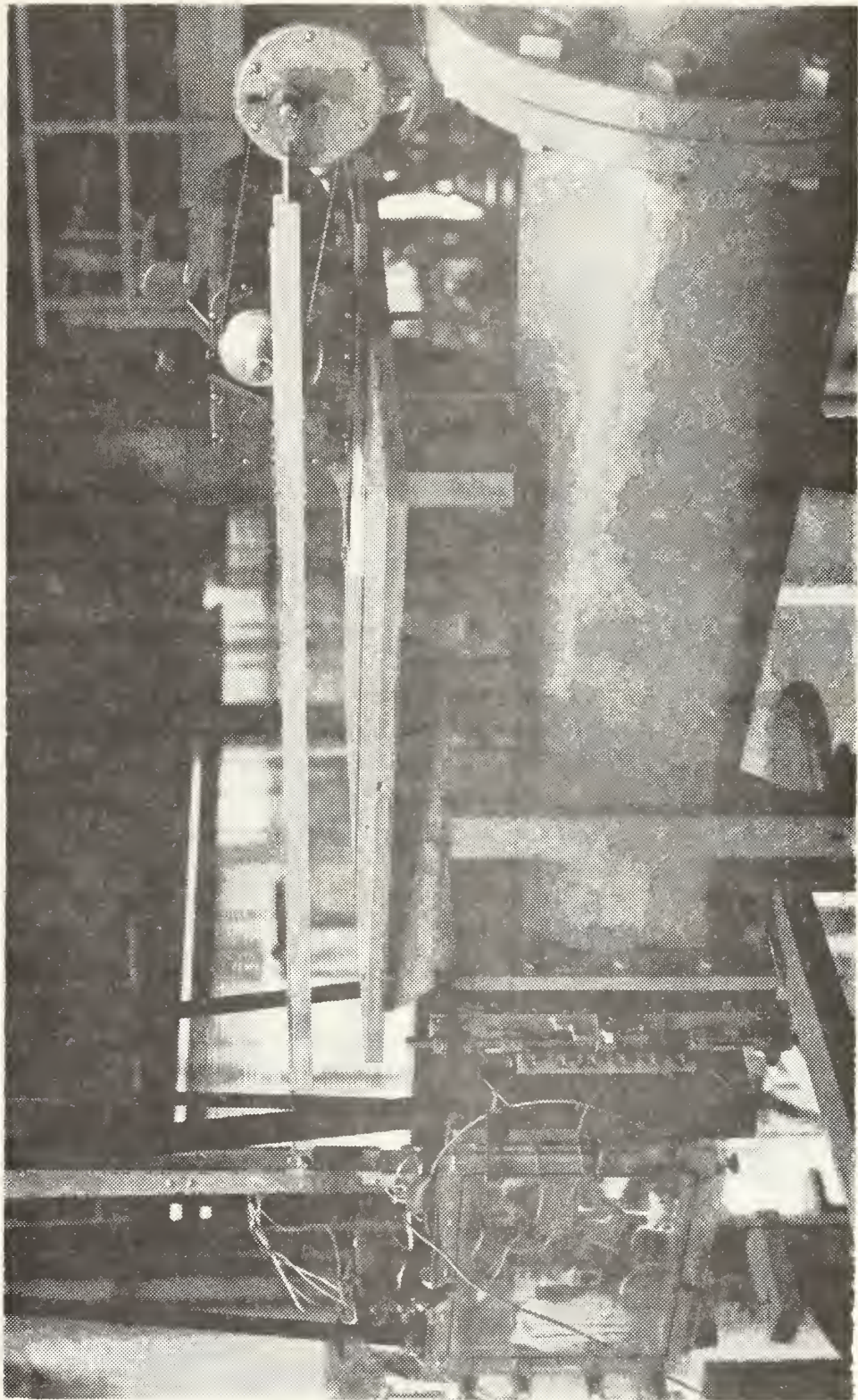
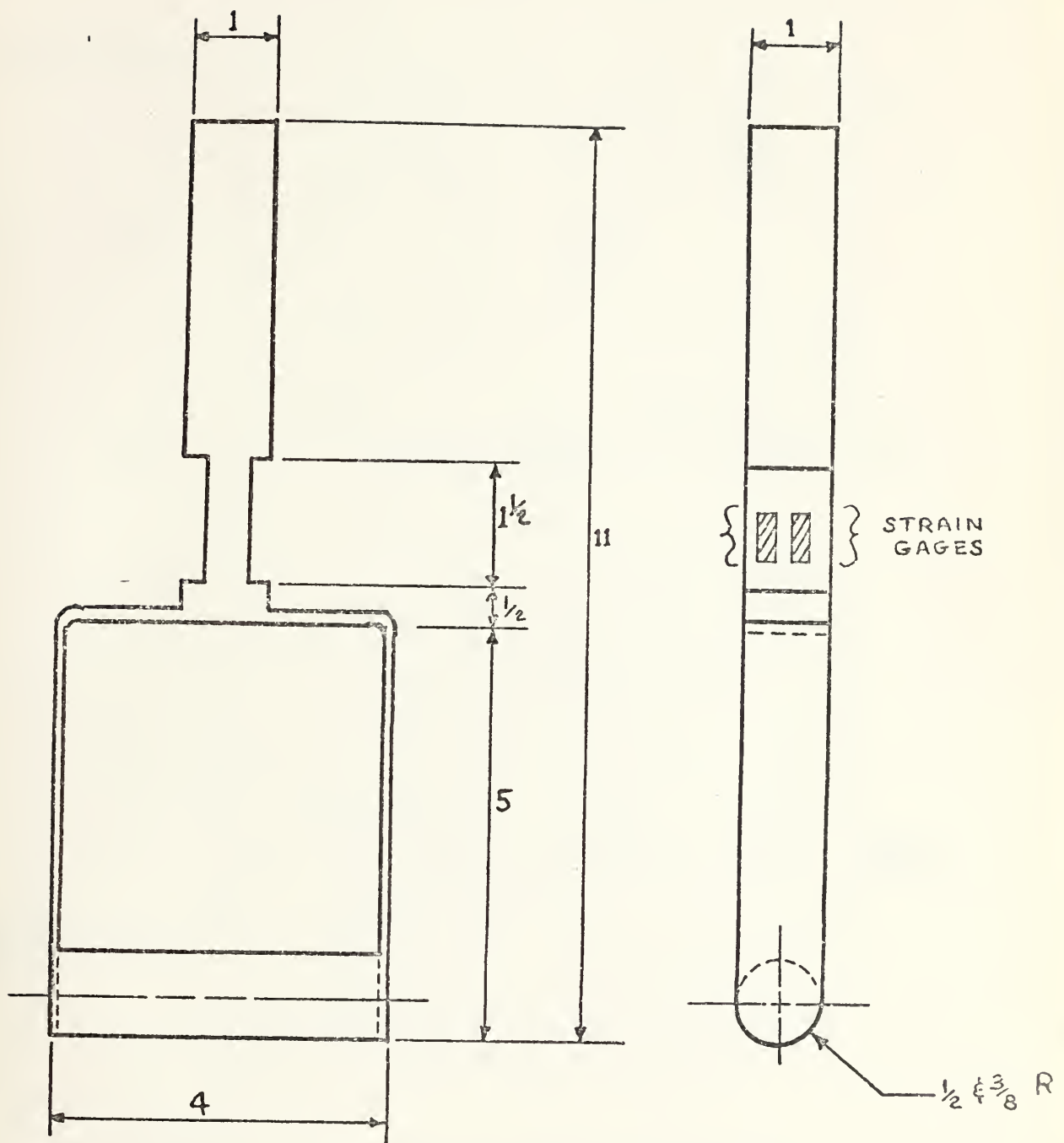


Figure 2. Electric motor and crank arrangement.







SCALE = ONE HALF

DIMENSIONS: INCHES

Figure 3. Aluminum yoke/force transducer



$D = 1.0$  INCHES

$A/D = 0.5$

$T = 0.485$  sec

IN-LINE FORCE

WET

DRY

ACCELERATION

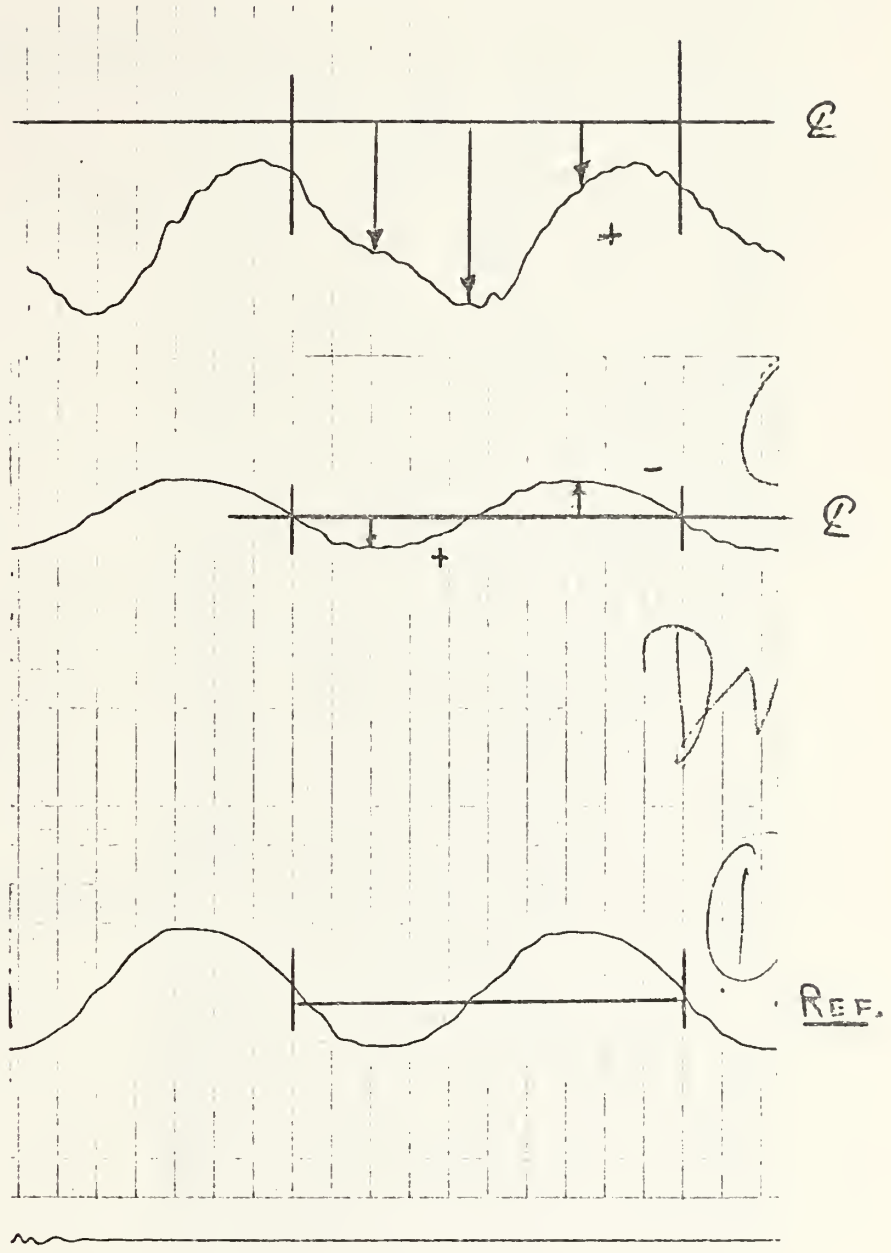


Figure 4. Force and acceleration traces.



$D = 1.0$  INCHES  
 $A/D = 0.5$   
 $T = 0.162$  sec

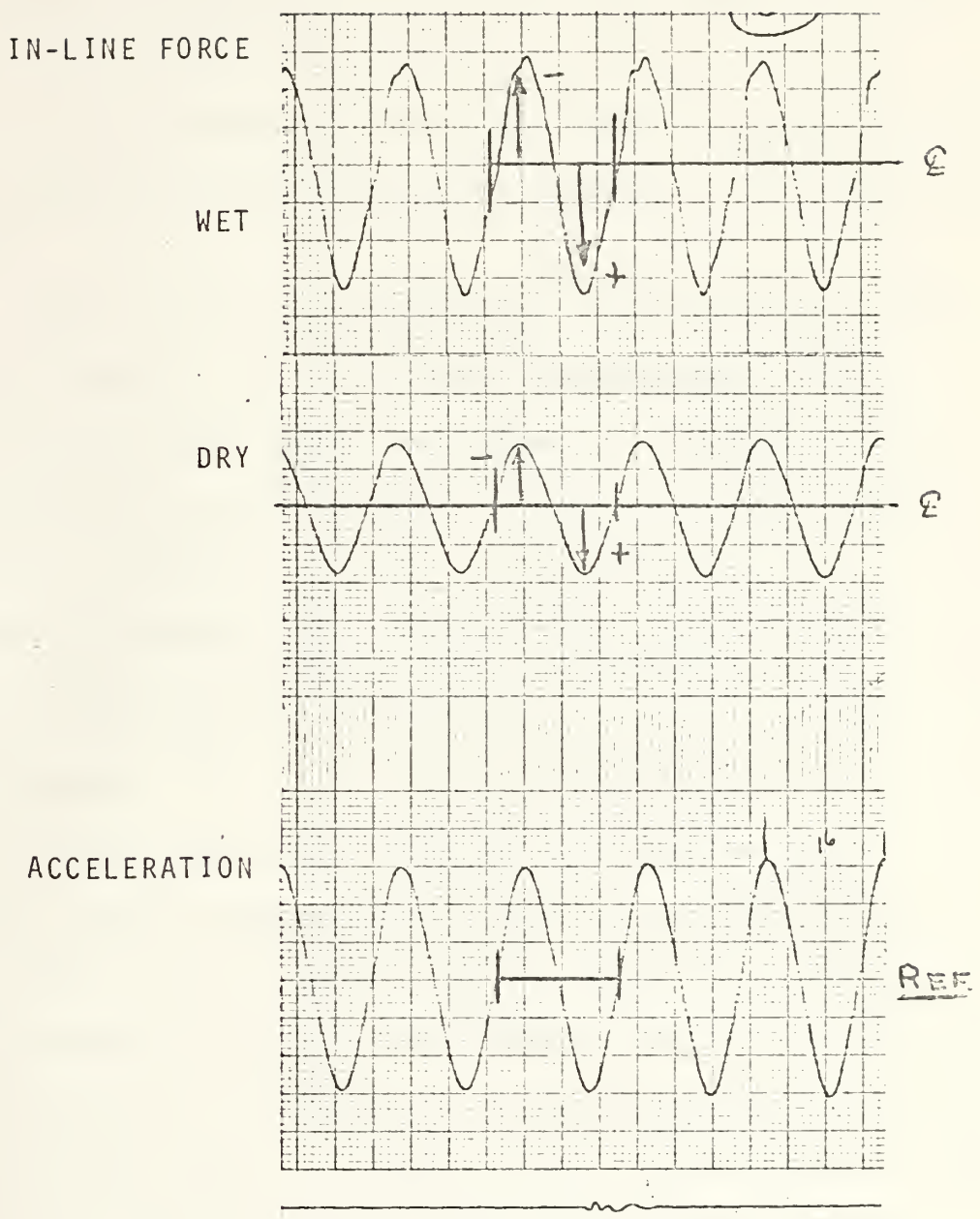


Figure 5. Force and acceleration traces.



#### 4. Sensor and Sensor Support

The mean velocity of the fluid was derived from the pressure differential of a pitot tube located upstream of the test specimen.

The instantaneous acceleration of the pendulum arm was monitored by an accelerometer attached to the arm of the pendulum.

The in-line force-measuring device consisted of a cantilever beam as part of the yoke arrangement. Four piezoresistive strain gages were mounted on the cantilever beam and properly waterproofed. A drawing of this force transducer is shown in Figure 3. The transducer was calibrated by hanging loads at the mid section of the cylinder in the vertical direction.

#### B. PROCEDURE

It was realized, during the early stages of the experiment, that a procedure of recording both the wet and dry oscillations at a common frequency would be necessary for a complete evaluation of the data. The wet oscillating force was made up of an inertial force and a resistive force produced by the fluid. In order to separate the forces, the dry force was recorded at the same frequency as the wet force and then subtracted from the wet force to give the pure resistive force.





To accomplish the physical recording of the wet and dry force oscillations, the water level in the tunnel was raised and lowered for each frequency tested. The proper status of the cylinder was noted on the chart recordings.

Figures 4 and 5 show the wet and dry forces as recorded for two typical frequencies. The acceleration curve was used as a reference in matching the wet and dry run curves. It was convenient that the acceleration traces were nearly perfect sine curves.

From traces similar to the one shown in Figure 4, the in-line force was read and recorded from both the wet and dry curves. The result was then punched on IBM computer cards for every 1.0 or 2.0 mm's.

Finally the drag and inertia coefficients were calculated through the computer program given in Appendix A.



### III. DISCUSSION OF RESULTS

The results of the present investigation will be reported in terms of  $\bar{C}_d$ ,  $C_d$ , and  $C_m$  as a function of  $D/\bar{V}T$  or  $\bar{V}T/D$  for various values of  $A/D$ . Figures 6 through 9 show  $\bar{C}_d$  as a function of  $D/\bar{V}T$  for  $A/D = 0.25, 0.5, 1.0$ , and  $1.5$ . In all cases  $\bar{C}_d$  is equal to its steady-state value for  $D/\bar{V}T = 0$ , i.e., for  $f = 0$ . The mean-drag coefficient reaches a maximum value at  $D/\bar{V}T$  between 0.2 and 0.35. The value of  $D/\bar{V}T$  at which this maximum occurs shifts from 0.35 to 0.2 as  $A/D$  increases. Furthermore, the maximum value of  $\bar{C}_d$  increases with increasing  $A/D$ . For example, for  $A/D = 1.5$  and  $D/\bar{V}T = 0.1$  (a wave of 15 ft amplitude and 20 second period superimposed on a current of 5 ft/sec velocity acting on a 10 ft diameter cylinder), the mean drag acting on a cylinder is increased by nearly 50%

Upon reaching a maximum,  $\bar{C}_d$  decreases sharply and as evidenced by the results of  $A/D = 0.25$ , reaches a value nearly equal to that corresponding to a steady flow at the velocity  $\bar{V}$ . Thus, for a given set of  $\bar{V}$  and  $D$ , in-line oscillations of either very small or very large frequencies do not alter the mean resistance, e.g., for  $A/D = 0.25$ ,  $\bar{C}_d$  remains essentially constant in the range  $0.45 < D/\bar{V}T < 0.1$  or in the range  $2 > \bar{V}T/D > 10$ . In other words, when the frequency  $1/T = f$  exceeds twice the Strouhal frequency, there is no more vortex synchronization.



From a practical point of view, the increase of  $\bar{C}_d$  to values several times larger than the steady-state value is quite significant. Field studies conducted to determine wave forces on cylinders may be strongly affected by the presence of mean currents and may yield drag and inertia coefficients which are significantly different from those obtained with purely harmonic oscillations. It is, of course, fully realized that the ocean waves are not usually simple harmonic, but are rather irregular.

Figures 6 through 9 also show two additional facts of special importance. Firstly, as far as  $\bar{C}_d$  is concerned, there is no special significance of the case where  $U_m > \bar{V}$ . A simple calculation shows that  $U_m > \bar{V}$  for  $D/\bar{V}T > 0.637$  for  $A/D = 0.25$ ; for  $D/\bar{V}T > 0.318$  for  $A/D = 0.5$ ; for  $D/\bar{V}T > 0.159$  for  $A/D = 1.0$ ; and  $D/\bar{V}T > 0.106$  for  $A/D = 1.5$ . Secondly,  $\bar{C}_d$  is not equal to  $C_d$  (steady)  $(1 + 0.5U_m^2/\bar{V}^2)$ . In fact, only for very small values of  $U_m/\bar{V}$  that the time-averaged drag force can be calculated by increasing the force calculated from the steady flow by a factor  $(1 + 0.5U_m^2/\bar{V}^2)$ .

The Fourier-averaged drag and inertia coefficients  $C_d$  and  $C_m$  are shown in Figures 10 through 13 and 14 through 17 respectively as function of  $\bar{V}T/D$  for  $A/D = 0.25, 0.5, 1.0$ , and  $1.5$ . The coefficients  $C_d$  and  $C_m$  have also been calculated through the use of the least-squares method and a modified least-squares method. Since the results differ very little from those obtained through the use of the Fourier analysis,



only the Fourier-averaged coefficients are presented herein. On each graph, the values corresponding to  $\bar{V}T/D = 0$  are taken from Sarpkaya [14].

Firstly, it is noted that both  $C_d$  and  $C_m$  exhibit larger scatter than  $\bar{C}_d$ . The reasons for this scatter are rather understandable. For a given  $A$ ,  $D$ , and  $\bar{V}$ ,  $A/D$  is fixed and only  $T$  is variable in  $\bar{V}T/D$ . Thus, very small values of  $\bar{V}T/D$  correspond to very small values of  $T$  or to rather high frequencies of oscillation. This means that not only the fluid forces but also the inertial forces due to the mass of the oscillating system are large. Furthermore, at very high frequencies, the acceleration is not exactly harmonic and the second order accelerations may introduce some error into the evaluation of the data. In passing, it should be noted that attempts to increase  $T$  by decreasing  $\bar{V}$  so as obtain the same value of  $\bar{V}T/D$  are not very desirable since this leads to smaller Reynolds numbers. This and similar other facts simply point out the extreme difficulty of experimentation in unsteady flows and are sufficient to put the critical reader in touch with his wisdom.

For large values of  $\bar{V}T/D$ , the frequencies are very low and give rise to rather small fluctuating forces about the mean drag force. Thus, the decomposition of the small alternating component of the total force into in-phase and out-of-phase components, i.e., into drag and inertial forces, leads to a number of difficulties. In this range, small





errors in phase between, say, zero acceleration and the actual position of the cylinder in the cycle lead to relatively large errors in the evaluation of  $C_d$  and  $C_m$ . In spite of this, however, the data are sufficiently consistent to draw quantitative conclusions. It is apparent from Figures 10 through 13 that  $C_d$  increases rapidly with increasing values of  $\bar{V}T/D$  and remains positive throughout most of the  $\bar{V}T/D$  values. Only for  $A/D = 0.25$  and  $0.50$  and for very small values of  $\bar{V}T/D$  that  $C_d$  becomes negative. This range of  $\bar{V}T/D$  values nearly fall in the velocity range where  $U_m > \bar{V}$ . Evidently, the cylinder cannot extract energy from the fluid when  $C_d > 0$ . Thus, the negative values of  $C_d$  in a small region of  $\bar{V}T/D$  values imply that it is possible to excite in-line oscillations of very small amplitudes at frequencies five or more times the Strouhal frequency. Such an oscillation has not previously been observed partly because no one has excited the elastic test cylinders at such high frequencies and partly because the cylinders were allowed to oscillate in all directions with the consequence that in-line oscillations may have been obscured by relatively larger transverse oscillations.

The inertia coefficient  $C_m$  is shown in Figures 14 through 17 as a function of  $\bar{V}T/D$  for various values of  $A/D$ . For small values of  $\bar{V}T/D$ ,  $C_m$  approaches its ideal potential-flow value of unity. Near Strouhal frequency, however,  $C_m$  decreases sharply, particularly for small amplitudes of



oscillation. In other words, near Strouhal frequency, the force acting on a cylinder undergoing small amplitude in-line oscillations is primarily from drag. This is rather interesting if we consider the fact that for small values of  $A/D$ , say  $A/D = 0.25$  or  $U_m T/D = 1.57$ , the force acting on a cylinder oscillating in a fluid otherwise at rest (i.e.,  $\bar{V} = 0$ ) is essentially of inertial nature [14]. Thus, the presence of a mean flow superimposed on harmonic oscillations can significantly alter not only the magnitude of the inertial force but also the nature of the entire flow structure about the body. For large values of  $\bar{V}T/D$  where  $U_m/\bar{V}$  is rather small ( $U_m/\bar{V} = 0.15$  for  $A/D = 0.25$ , and  $U_m/\bar{V} = 0.30$  for  $A/D = 0.50$ ), the forces acting on the body follow the changes in velocity, as noted in connection with the discussion of  $C_d$ , and the inertial component of the force is negligibly small. However, it must be emphasized once again that the quasi-steady model is grossly inadequate to predict the average drag and the response of the instantaneous force to changes in velocity for large periods or for extremely small frequencies bears no relation to the existence of an actual quasi-steady state of flow.



#### IV. CONCLUSIONS

The experimental investigation of the in-line oscillations of a circular cylinder in a flow with a mean velocity  $\bar{V}$  has yielded the force coefficients  $\bar{C}_d$ ,  $C_m$ , and  $C_d$  and has shown that:

(a) The mean flow has significant effects on  $\bar{C}_d$ ,  $C_m$  and  $C_d$  and that the results of the experiments with harmonic oscillations in a fluid otherwise at rest are not applicable to oscillations of a cylinder in uniform flow;

(b) In a region of small values of  $D/\bar{V}T$  where the frequency of oscillations is five or more times larger than the Strouhal frequency,  $C_d$  is negative and energy may be transferred from the fluid to the cylinder. This, in turn, may give rise to self-excited in-line oscillations in a manner similar to those for the transverse oscillations; and that

(c) The most significant region of the  $D/\bar{V}T$  values in which the force coefficient differ significantly from those obtained in steady flow or with harmonic oscillations in a fluid at rest occur in the vicinity of the Strouhal numbers.



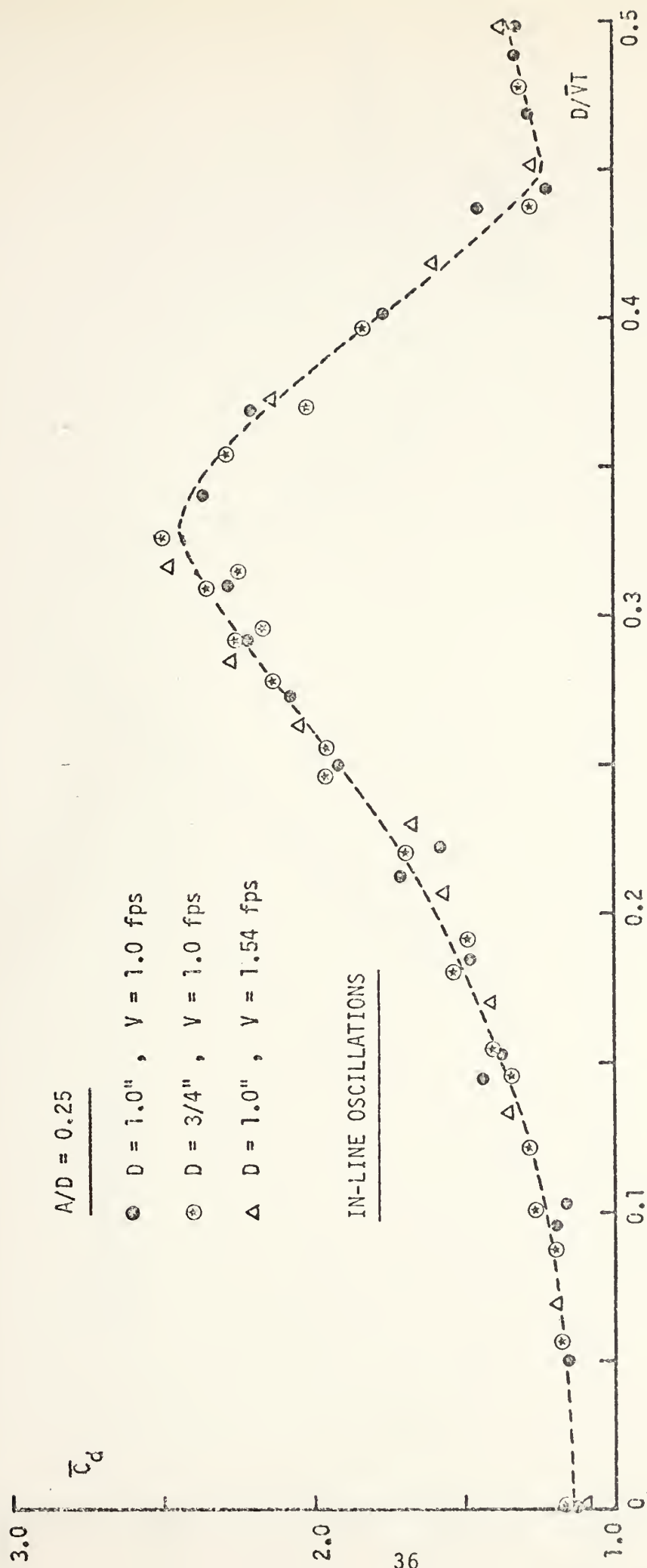


Figure 6. Mean drag coefficients versus  $D/\sqrt{VT}$  for  $A/D = 0.25$ .







Figure 7. Mean drag coefficients versus  $D/\sqrt{VT}$  for  $A/D = 0.50$ .





Figure 8. Mean drag coefficients versus  $D/\sqrt{VT}$  for  $A/D = 1.00$ .



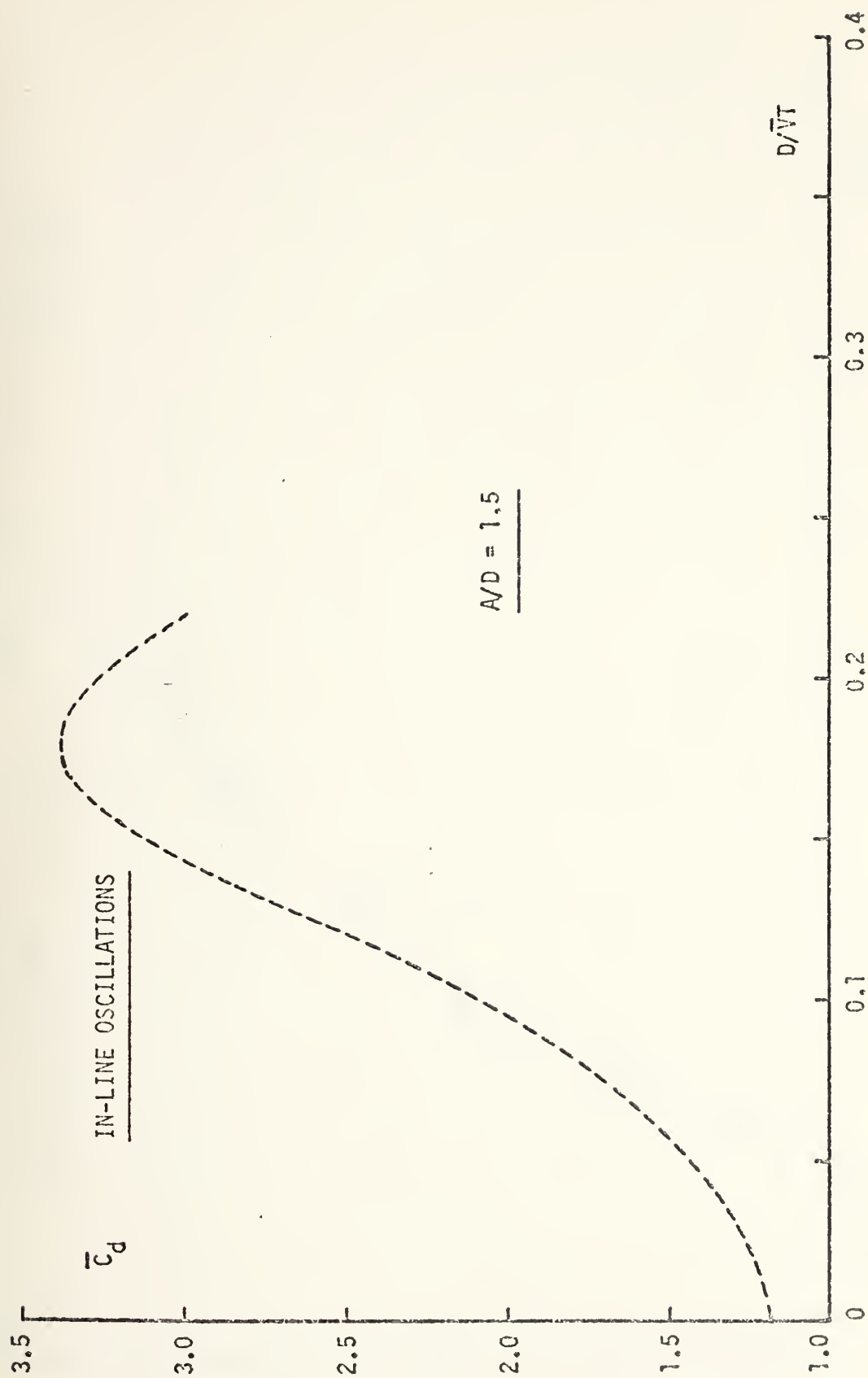


Figure 9. Mean drag coefficients versus  $D/\sqrt{VT}$  for  $A/D = 1.50$ .





Figure 10. Fourier-Averaged drag coefficient versus  $V_T/D$  for  $A/D = 0.25$ .





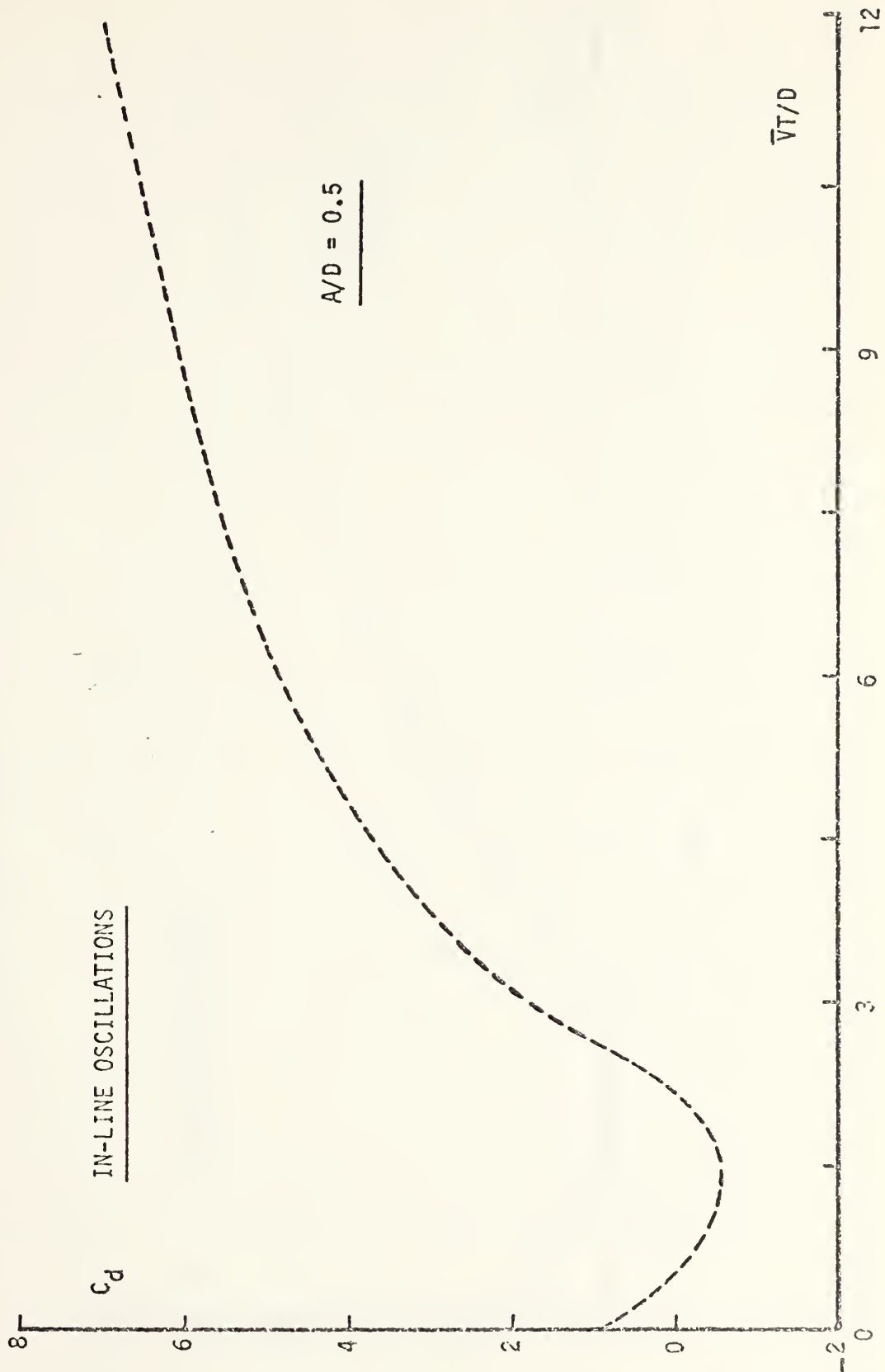


Figure 11. Fourier-Averaged drag coefficient versus  $\bar{V}T/D$  for  $A/D = 0.50$ .



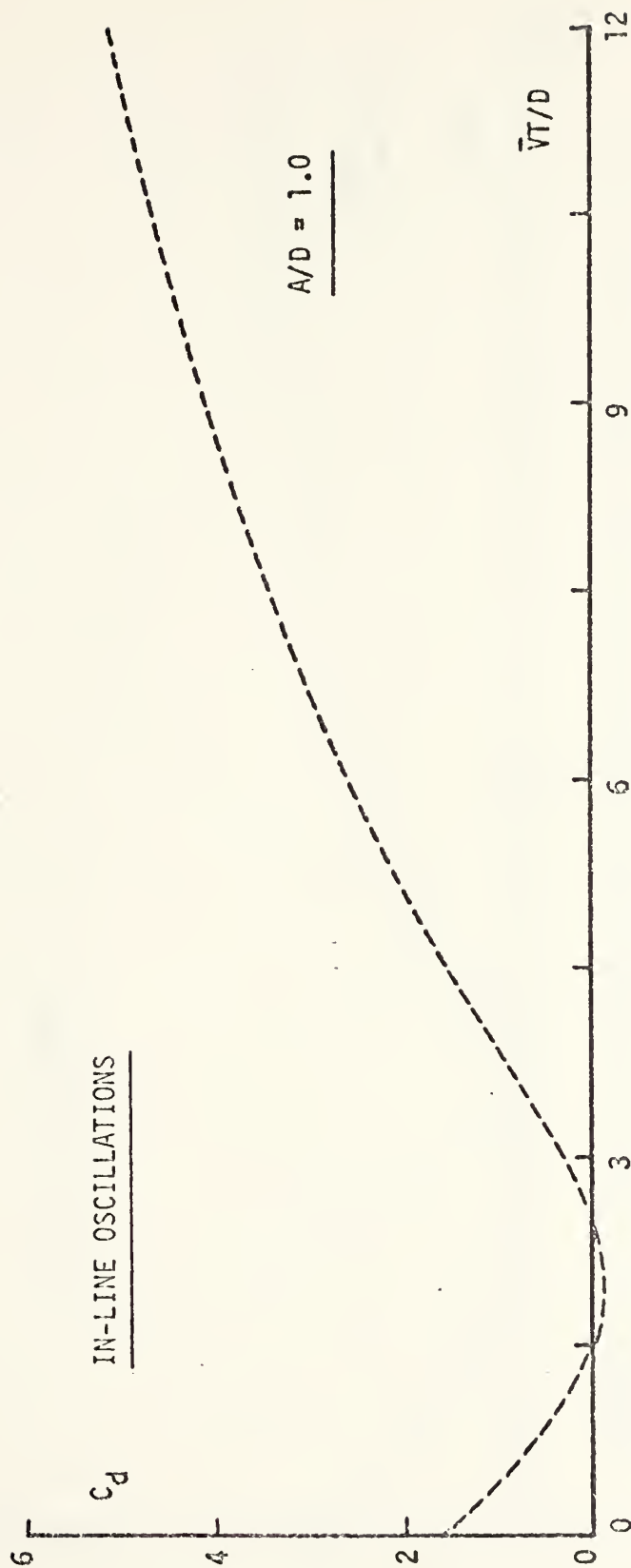


Figure 12. Fourier-Averaged drag coefficient versus  $\bar{V}T/D$  for  $A/D = 1.00$ .



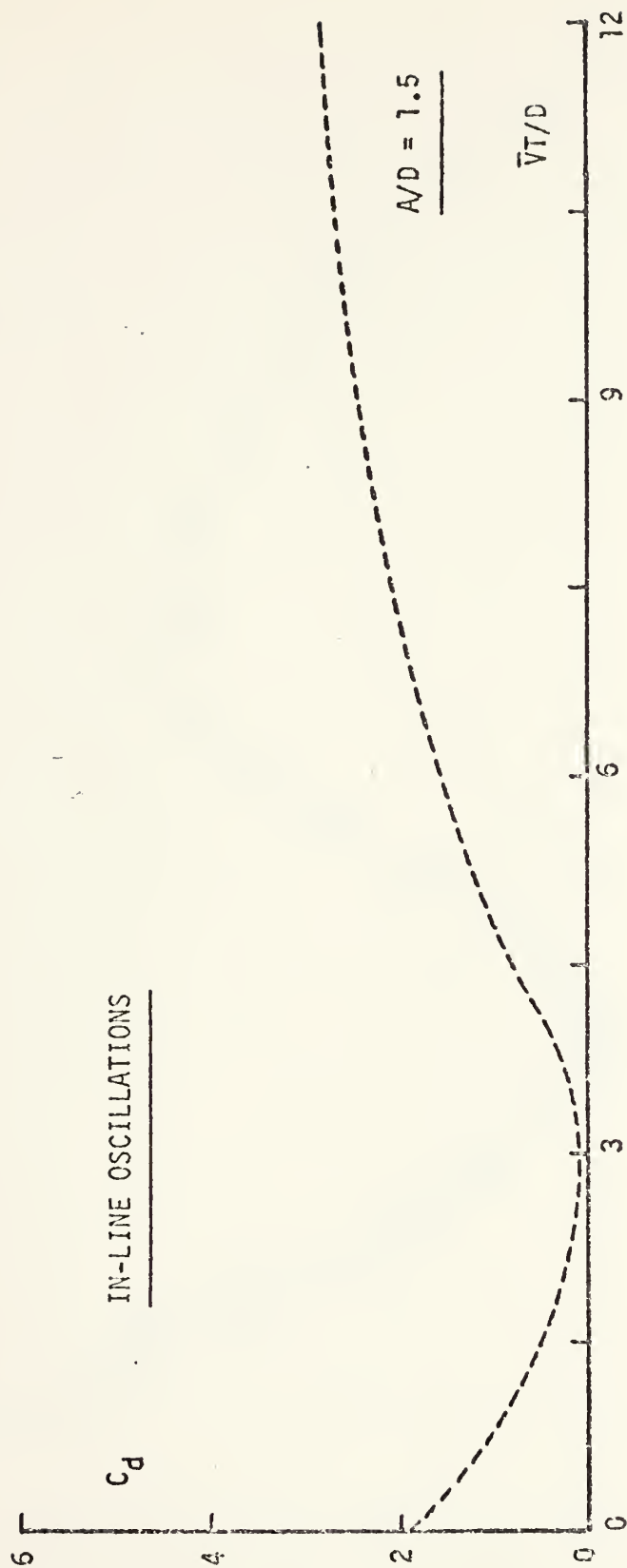


Figure 13. Fourier-Averaged drag coefficient versus  $\bar{V}_T/D$  for  $A/D = 1.50$ .



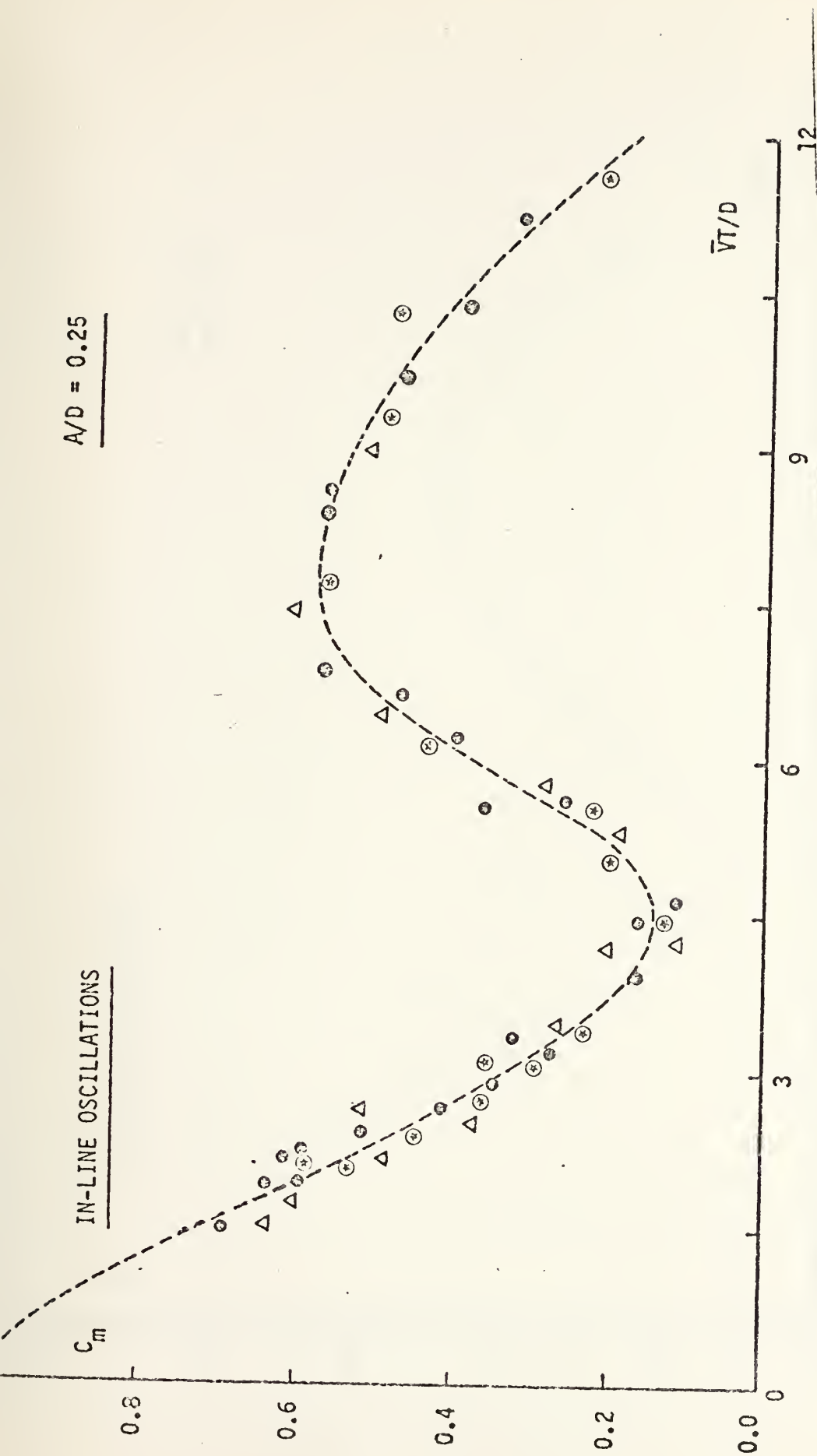


Figure 14. Fourier-Averaged inertia coefficient versus  $\bar{T}/D$  for  $A/D = 0.25$ .





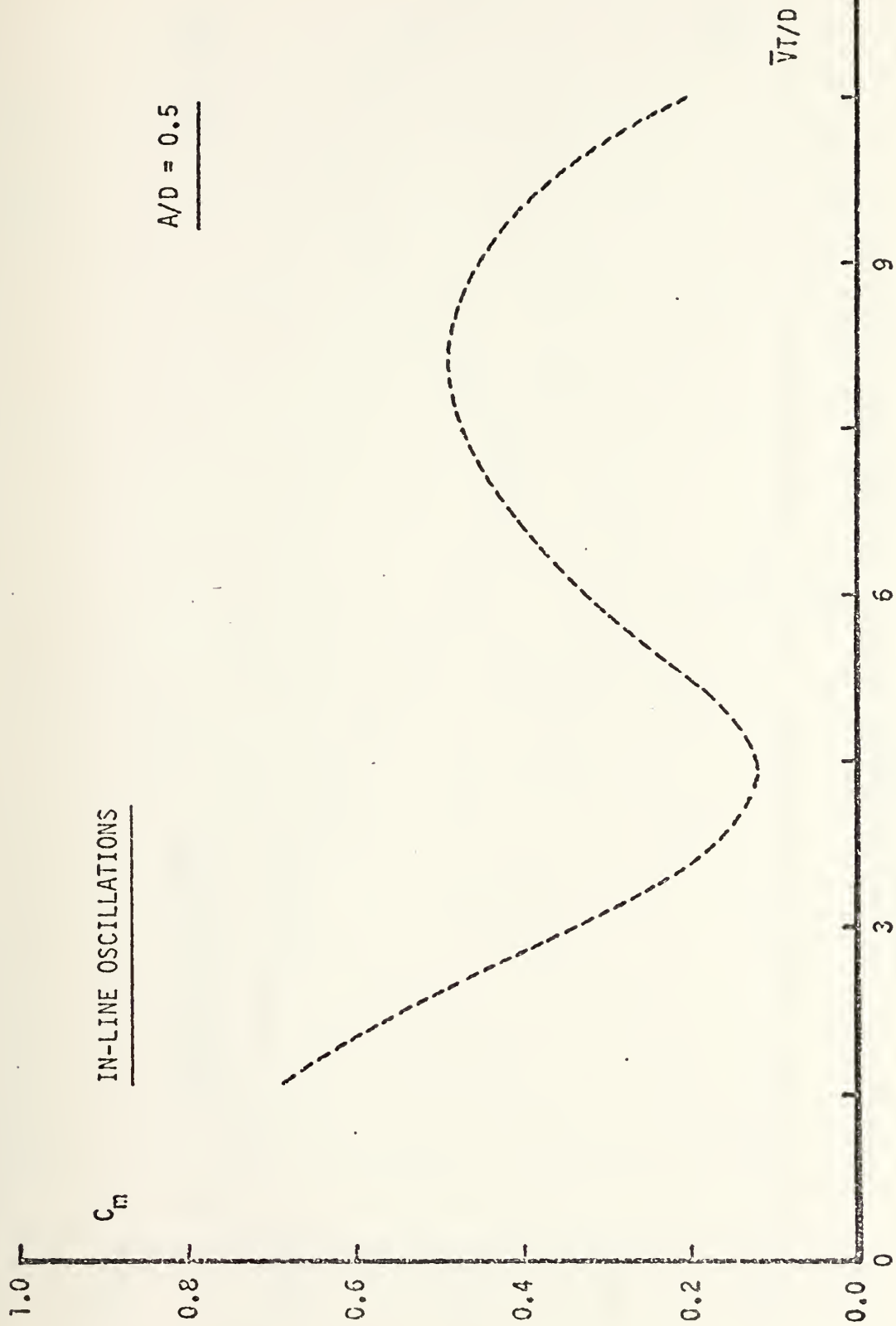


Figure 15. Fourier-Averaged inertia coefficient versus  $\bar{V}_T/D$   
for  $A/D = 0.50$ .



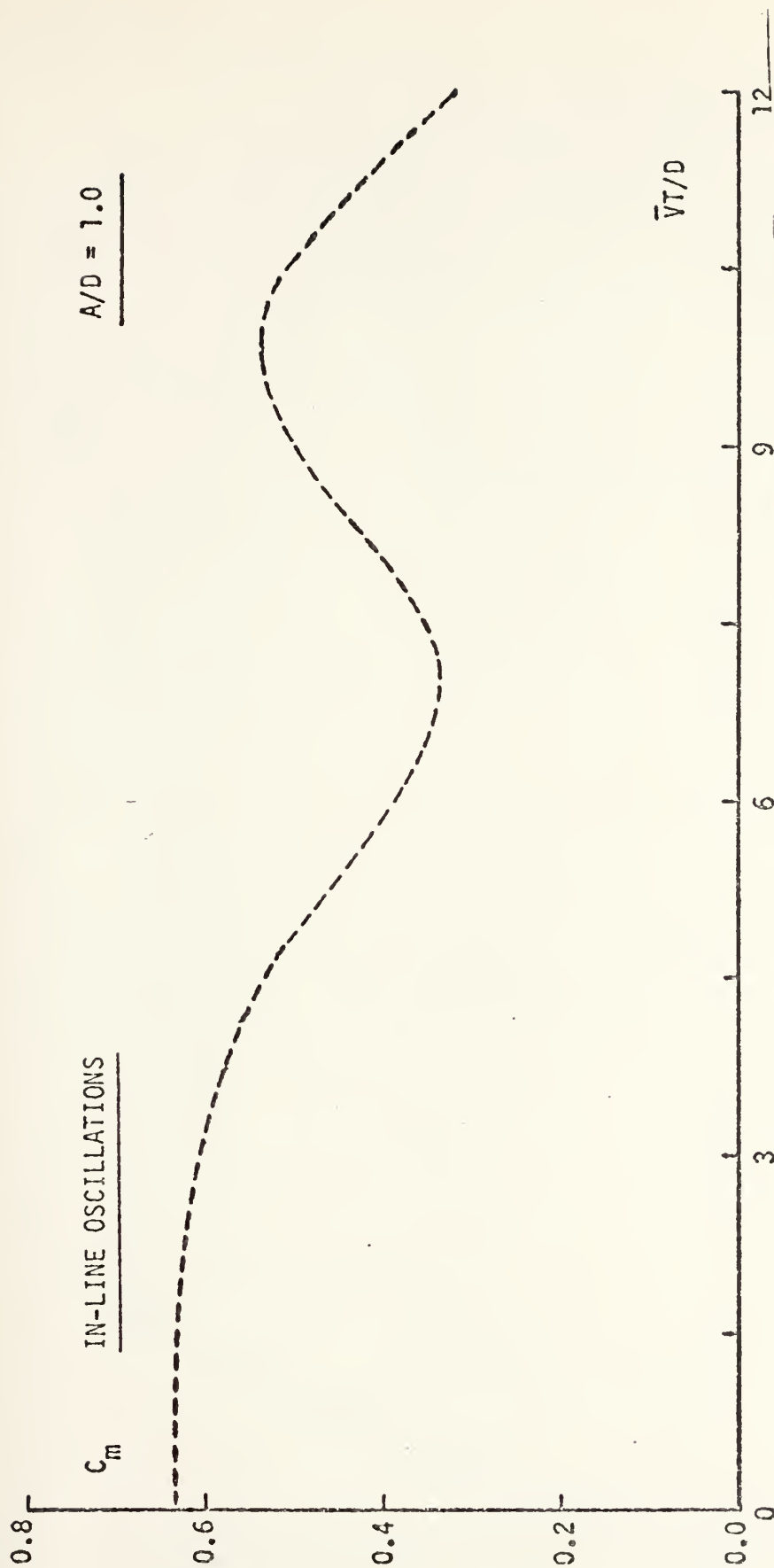


Figure 16. Fourier-Averaged inertia coefficient versus  $\bar{V}_T/D$   
for  $A/D = 1.00$ .



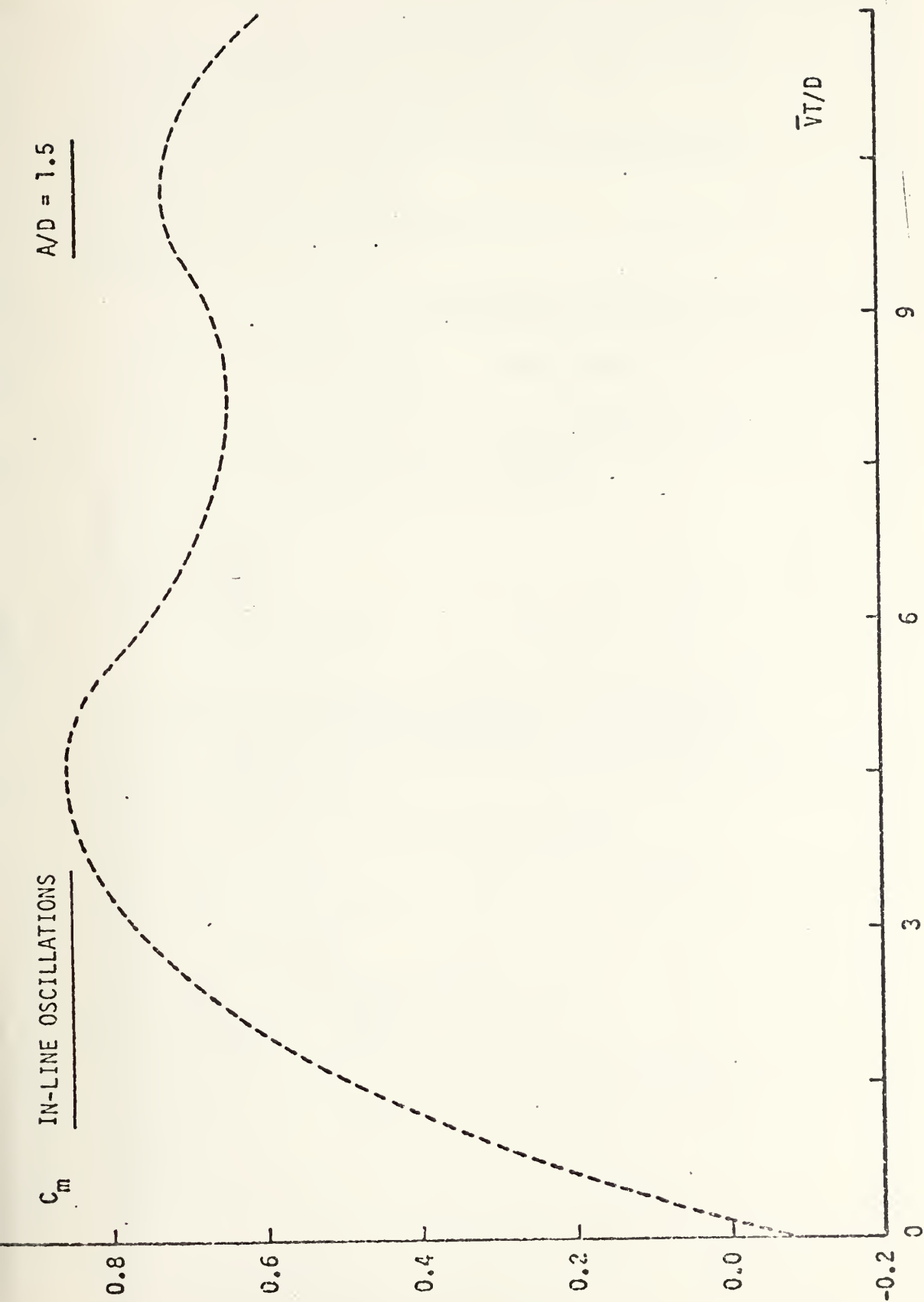


Figure 17. Fourier-Averaged inertia coefficient versus  $\bar{V}_T/D$  for  $A/D = 1.50$ .



# APPENDIX A

## COMPUTER PROGRAMS

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      AVERAGE CD AND CM CALCULATIONS FOR CYLINDERS
C      TIME=DIMENSIONLESS TIME (TIME/PERIOD)
C      BETA=DIMENSIONLESS DISPLACEMENT(UMAX*PER/DIA)
C      CL=CYLINDER LENGTH IN FEET
C      DIA=DIAMETER OF CYLINDER IN FEET
C      AMP=AMPLITUDE OF MOTION IN FEET
C      PER=PERIOD IN SECS (CHART PERIOD/CHART SPEED)
C      Z1=FORCE COEFFICIENTS
C      CM=INERTIA COEFFICIENT
C      CD=DRAG COEFFICIENT
C      REMF=REMAINDER FUNCTION
C      A1,B1=FOURIER COEFFICIENTS
C      N=NUMBER OF DATA SETS
C      CN=CONVERSION FACTOR
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      DIMENSION FORCE(100)
C      READ IN PARAMETERS WHICH CHARACTERIZE THE DATA SET
C
C      N = 12
C      DO 100 I=1,N
C      READ(5,10) DIA,AMP,PER,NCARD,UMX,CN,FMM,NRUN
C
C      READ(5,55) (FORCE(K), K=1,NCARD)
C      G=32.174
C      RHO=62.4/G
C      PI=3.14159
C      CNU=0.000105
C      CL=0.3323334
C      VBAF=0.9569
C
C      INITIATE DATA SETS
C
C      COMPUTE FORCE COEFFICIENTS
C      Z1=(2*PER*PER)/(PI*PI*PI*DIA*CL*RHO*AMP)
C      Z2=(-3*PER*PER)/(8*RHO*DIA*PI*CL*AMP*AMP)
C      Z2LS=(-4*PER*PER)/(3*PI*PI*RHO*DIA*CL*AMP*AMP)
C      COMPUTE BETA
C      BETA=2*PI*AMP/DIA
C      REYNQ=(VBAF*DIA)/CNU
C      WRITE(6,15) DIA,AMP,PER,UMX,CN,NRUN
C      WRITE(6,20)
C      DO 65 K=1, NCARD
C      SUM =0.0
C      SUM =SUM + FORCE(K)
65  CONTINUE
C      CORR = SUM/NCARD
C      DO 85 K=1, NCARD
C      FORCE(K) = FORCE(K) - CORR
C      FORCE(K) = - FORCE(K)
85  CONTINUE
C      FMM = FMM - CORR
C      FMM = -FMM
C      TIME=0.0
C      CM=0.0
C      CD=0.0
C      CMLS=0
C      CCLS=0
C      FAA=0.0
C      FBB=0.0
C      FCC=0.0
C      FDD=0.0
C      FEE=0.0
C      DELTAT=1.0/NCARD
C      J=NCARD-1
C      DO 200 K=1,J
C      F=CN*(FORCE(K)+FORCE(K+1))/2.0
C      ALPHA=2*PI*(TIME+DELTAT/2.0)
C      SINA=SIN(ALPHA)

```





```

COSA=COS (ALPHA)
FSINA=      F*SINA
FCOSA=      F*COSA
FLS=        F*COSA*ABS(COSA)
FLS=        F*CDLS*ABS(COSA)
F/AS=2*PI*F*1.0000*COSA*COSA*COSA*COSA*F
FBBS=2*PI*F*F*COSA*ABS(COSA)*1.000*F
FCCS=2*PI*F*F*SINA*COSA*ABS(COSA)*1.0000*F
FDDS=2*PI*F*F*SINA*SINA*1.0000*F
FEES=2*PI*F*F*SINA*1.0000*F
FAA=FAAS+FAA
FBB=FBS+FBBS
FCC=FCCS+FCC
FDD=FDDS+FDD
FEE=FEES+FEE
CM=FSINA+CM
CD=FCOSA+CD
CMLS=FSINA+CMLS
CDLS=FLS+CDLS
WRITE(6,30)TIME,ALPHA,COSA,SINA,F,FCOSA,FSINA
TIME=TIME+DELTAT

```

200

```

CONTINUE
TIME=TIME+DELTAT
F=CM*(FURCE(1)+FORCE(NCARD))/2.0
ALPHA=2*PI*(TIME+DELTAT/2.0)
SINA=SIN(ALPHA)
COSA=COS(ALPHA)
FSINA=      F*SINA
FCOSA=      F*COSA
FAAS=2*PI*F*1.0000*COSA*COSA*COSA*COSA*F
FBBS=2*PI*F*F*COSA*ABS(COSA)*1.000*F
FCCS=2*PI*F*F*SINA*COSA*ABS(COSA)*1.0000*F
FDDS=2*PI*F*F*SINA*SINA*1.0000*F
FEES=2*PI*F*F*SINA*1.0000*F
FAA=FAAS+FAA
FBB=FBS+FBBS
FCC=FCCS+FCC
FDD=FDDS+FDD
FEE=FEES+FEE
CM=FSINA+CM
CD=FCOSA+CD
CMLS=FSINA+CMLS
CDLS=FLS+CDLS
WRITE(6,30)TIME,ALPHA,COSA,SINA,F,FCOSA,FSINA
CM=Z1*CM*DELTAT
CD=Z2*CD*DELTAT
CMLS=Z1*CMLS*DELTAT
CDLS=Z2*CDLS*DELTAT
CMFF=(Z1/2.0)*(FEE*FAA-FCC*FBB)/(FDD*FAA-FCC*FCC)

```

```

CDEF=(-4.0*Z2/(3.0*PI))*(FEE*FCC-FDD*FBB)/(FDD*FAA-FCC*FCC)
WRITE(6,35)
WRITE(6,40)CM,CD,BETA,REYNO,CMLS,CDLS,CMFF,CDEF
ANGLE=0.0
WRITE(6,45)
TIME=0.0
DO 300 K=1,NCARD
F=CL*FORCE(K)
THETA1=((2.0*PI)/360)*ANGLE
C1=(ABS(COS(THETA1)))*COS(THETA1)
C2=RHQ*((UMX**2.0)/2.0)*DIA*CL
C3=((PI**2)*DIA*SIN(THETA1))/(UMX*PER)
F1=(CM*C3-CD*C1)
FLS=(CMLS*C3-CDLS*C1)
FFCP=(CMFF*C3-CDEF*C1)
F=F/C2
REMF=ABS(F)-ABS(F1)
FMAX=FMAX*CM/C2
REMF=REMF/FMAX
RLS=(ABS(F)-ABS(FLS))/FMAX
RFFC=(ABS(F)-ABS(FFCP))/FMAX
WRITE(6,50)TIME,F,F1,REMF,FLS,RLS,FFOR,KFOR,FMAX,C2
ANGLE=ANGLE+360.0/NCARD
TIME=TIME+DELTAT

```



```

300 CONTINUE
100 CONTINUE
10  FORMAT(3F8.4,I8,F8.4,F10.6,F8.4,I7)
15  FORMAT('1',2X,'C1A=',F8.4,2X,'AMP=',F8.4,2X,'PER=',
1F8.4,2X,'UNX=',F8.4,2X,'CN=',F8.4,2X,'NDUN=',I6)
20  FORMAT('0',3X,'TIME/PER',7X,'ALPHA',7X,'COS4',15X,'SINA
3',2X,'F',7X,'FCCSA',5X,'FSINA')
30  FORMAT('0',3F12.4,F19.4,3F12.4)
35  FORMAT('0',7X,'CM=',7X,'CD=',7X,'BETA=',7X,'REYNO=',7X,
1'CMLS=',7X,'CDLS=',7X,'CMFF=',7X,'CDFF=')
40  FORMAT('0',6F12.4,2F12.4)
45  FORMAT('0',7X,'INE',9X,'F',9X,'F1',9X,'PEMF',9X,'FLS',9X,
1'RLS',12X,'FFCR',12X,'RECR')
50  FORMAT('0',6F12.4,4F15.4)
55  FORMAT(F10.4)
STOP
END

```



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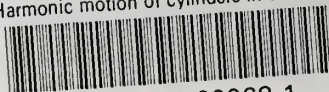
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